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2DOF CONTROLLERS TUNING METHOD FOR INTEGRATING PLANTS

Miluše Vítečková[1] **and Antonín Víteček**[2]

[1,2]VŠB – Technical University of Ostrava, Faculty of Mechanical Engineering
17. listopadu 15, CZ-708 33 Ostrava-Poruba, Czech Republic
[2]Kielce University of Technology, Poland
Tel.: +420 597 324 493 Fax: +420 59 691 6129
e-mail:miluse.viteckova@vsb.cz, antonin.vitecek@vsb.cz

Abstract

The article deals with a new 2DOF PI and PID controllers tuning method for integrating plants. The described approach is derived from the multiple dominant pole method and it enables the achievement of aperiodic servo and regulatory step responses without the overshoots.

Keywords: 2DOF controllers, PID, integrating plant, time delay

1 INTRODUCTION

The use of PI and PID controllers for integrating plants is not often discussed in the control system literature. At the same time their tuning does not belong among simple problems [Åström & Hägglund 2006; O´Dwyer 2006; Hudzovič & Kozáková 2001; Rosinová & Markech 2008]. It is given by the degree of the astatism $q \geq 2$, which induces a predisposition to oscillations and big overshoots [Åström & Hägglund 2006; Vítečková & Víteček 2007, 2008, 2009].

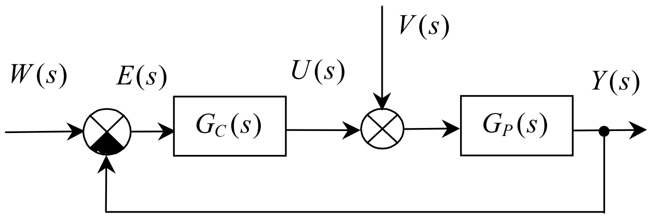

Fig. 1. Control system

The transfer functions for the control system in the Fig. 1 for errors caused by the desired and disturbance variables for $q \geq 2$ have the forms

$$G_{we}(s) = \frac{E(s)}{W(s)} = \frac{s^q M(s)}{N(s)} \tag{1}$$

and

$$G_{ve}(s) = \frac{E(s)}{V(s)} = -\frac{sM_P(s)}{N(s)} \tag{2}$$

where $M(s)$ and $M_P(s)$ are the polynomials or quasipolynomials, from which is not possible to point out the complex variable s; $N(s)$ – the characteristic polynomial or quasipolynomial of the control system; $E(s)$, $W(s)$, $V(s)$ and $Y(s)$ – the transforms of the error, the desired variable, the disturbance variable and the output variable; $G_C(s)$ – the controller transfer function; $G_P(s)$ – the plant transfer function.

For the step changes of the desired and disturbance variables

$$W(s)=\frac{w_0}{s},\ V(s)=\frac{v_0}{s} \tag{3}$$

where w_0 and v_0 are the step magnitudes, the transform of the control errors are given by the relations

$$E_w(s)=G_{we}(s)W(s)=\frac{s^{q-1}M(s)w_0}{N(s)} \tag{4}$$

and

$$E_v(s)=G_{ve}(s)V(s)=-\frac{M_P(s)v_0}{N(s)} \tag{5}$$

On the basis of the final value theorem there can be obtained

$$e_w(\infty)=\lim_{s\to 0}[sE_w(s)]=\lim_{s\to 0}\frac{s^{q}M(s)w_0}{N(s)}=0 \tag{6}$$

$$e_v(\infty)=\lim_{s\to 0}[sE_v(s)]=-\lim_{s\to 0}\frac{sM_P(s)v_0}{N(s)}=0 \tag{7}$$

That's just it the demand the zero steady state error caused by the disturbance step in the plant input it is necessary to use controllers with the integrating term, i.e. PI and PID controllers. The I controller by reason of the structural instability is generally unusable.

For the steps changes (3) the control areas are given by the relations

$$\int_0^\infty e_w(t)dt=\lim_{s\to 0}E_w(s)=\lim_{s\to 0}\frac{s^{q-1}M(s)w_0}{N(s)}=0 \tag{8}$$

and

$$\int_0^\infty e_v(t)dt=\lim_{s\to 0}E_v(s)=-\lim_{s\to 0}\frac{M_P(s)v_0}{N(s)}\neq 0 \tag{9}$$

The interpretation of the obtained relations is very important. From the first relation (8) follows that the control area is equal to zero, i.e. for the servo step response the course of the controlled variable cannot be without the overshoot, see Fig. 2 [Vítečková & Víteček 2007, 2008].

It means that it is impossible to obtain for the PI and PID controllers the aperiodic servo step response without the overshoot. If the overshoot is inadmissible, then it is necessary to use a limitation of the desired variable velocity or the input filtration of the desired variable. In this case it is very favourable to use the controllers with two degrees of freedom, where by the suitable choice of the weights of the desired variable in the proportional and derivative terms, it is possible to obtain corresponding input filtration [Åström & Hägglund 2006; Vítečková & Víteček 2007, 2008].

The interpretation of the second relation (9) is thereby the corresponding tuning of the PI and PID controllers is always possible to obtain the aperiodic regulatory response without the overshoot.

From the above mentioned it follows that the servo and regulatory step responses cannot be simultaneously aperiodic without overshoots for the integrating plants and the standard PI and PID controllers.

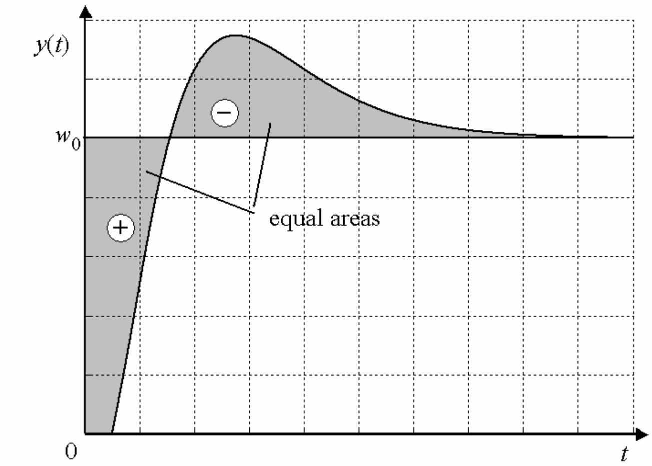

Fig. 2. Geometrical interpretation of relation (8)

It is obvious that the same conclusion holds for the integrating plants of arbitrary orders with a time delay too and the corresponding digital controllers as well [Krokavec & Filasová 2006].

Below the first order plants with integration will be considered, i.e. the degree of astatism q = 2 is supposed.

2 MULTIPLE DOMINANT POLE METHOD

In the paper the first order plant with integration without time delay is supposed

$$G_P(s) = \frac{k_1}{s(T_1 s + 1)} \tag{10}$$

and the standard PID controller

$$G_C(s) = r_0 + \frac{r_{-1}}{s} + r_1 s = r_0\left(1 + \frac{1}{T_I s} + T_D s\right) \tag{11}$$

$$T_I = \frac{r_0}{r_{-1}}, \quad T_D = \frac{r_1}{r_0} \tag{12}$$

where k_1 is the plant gain, T_1 – the time constant, r_0 – the proportional term weight (the controller gain), r_{-1} – the integral term weight, r_1 – the derivative term weight, T_I – the integral time, T_D – the derivative time.

It is obvious that for $r_1 = 0$ or $T_D = 0$ from (11) the transfer function of the PI controller

$$G_C(s) = r_0 + \frac{r_{-1}}{s} = r_0\left(1 + \frac{1}{T_I s}\right) \tag{13}$$

is obtained.

The multiple dominant pole method supposes the existence of a stable real dominant pole with the multiplicity increased by 1 over the number of the adjustable parameters of the chosen controller [Górecki 1971; Vítečková & Víteček 2007, 2008, 2009].

The multiple dominant pole and the adjustable parameter values can be obtained by solving the equation system

$$\frac{\mathrm{d}^i N(s)}{\mathrm{d}s^i} = 0 \tag{14}$$

where $N(s)$ is the characteristic polynomial of the control system with the plant (10) and chosen controller. For the plant (10) and the PID controller (11) is obtained

$$\begin{aligned} N(s) &= T_1 s^3 + (1 + k_1 r_1)s^2 + k_1 r_0 s + k_1 r_{-1} \\ \frac{\mathrm{d}N(s)}{\mathrm{d}s} &= 3T_1 s^2 + 2(1 + k_1 r_1)s + k_1 r_0 \\ \frac{\mathrm{d}^2 N(s)}{\mathrm{d}s^2} &= 6T_1 s + 2(1 + k_1 r_1) \end{aligned} \tag{15}$$

By solving that equation system (15) it may be obtained

$$\left.\begin{aligned} N(s) = 0 \\ \frac{\mathrm{d}N(s)}{\mathrm{d}s} = 0 \\ \frac{\mathrm{d}^2 N(s)}{\mathrm{d}s^2} = 0 \end{aligned}\right\} \Rightarrow \begin{aligned} s_3 &= -\frac{1 + k_1 r_1}{3T_1} \\ r_0 &= \frac{(1 + k_1 r_1)^2}{3k_1 T_1} \\ r_{-1} &= \frac{(1 + k_1 r_1)^3}{27 k_1 T_1^2} \end{aligned} \tag{16}$$

Since only the three equations are available, the triple dominant pole s_3, the weights r_0 and r_{-1} are dependent on the weight r_1. Therefore the ratio

$$\frac{T_D}{T_I} = \alpha = \frac{r_1 r_{-1}}{r_0^2} \ge 0 \tag{17}$$

is considered, from which it is obtained

$$r_1 = \alpha \frac{r_0^2}{r_{-1}} \tag{18}$$

and after substitution the last two relations from (16) in (18) it is obtained

$$r_1 = \frac{3\alpha}{(1 - 3\alpha)k_1}, \quad 0 \le \alpha < \frac{1}{3} \tag{19}$$

Now the triple dominant pole and weights can be expressed on the dependency of the ratio (17)

$$s_3^* = -\frac{1}{3T_1(1 - 3\alpha)}, \quad 0 \le \alpha < \frac{1}{3} \tag{20}$$

$$r_0^* = \frac{1}{3k_1 T_1 (1 - 3\alpha)^2} \tag{21}$$

$$r_{-1}^* = \frac{1}{27 k_1 T_1^2 (1 - 3\alpha)^3} \tag{22}$$

$$r_1^* = \frac{3\alpha}{k_1(1-3\alpha)} \tag{23}$$

or after consideration of the relations (12)

$$T_I^* = 9T_1(1-3\alpha) \tag{24}$$

$$T_D^* = 9\alpha T_1(1-3\alpha) \tag{25}$$

It is obvious, that for $\alpha = 0$ it is obtained tuning formulas for the PI controller (13).

The control system transfer function for the plant (10), the PID controller (11) and the computed controller adjustable parameters have the form

$$G_{wy}(s) = \frac{Y(s)}{W(s)} = \frac{G_C(s)G_P(s)}{1+G_C(s)G_P(s)} = \\ = \frac{\left[\frac{9}{2}T_1(1-3\alpha)\left(1+\sqrt{1-4\alpha}\right)s+1\right]\cdot\left[\frac{9}{2}T_1(1-3\alpha)\left(1-\sqrt{1-4\alpha}\right)s+1\right]}{[3T_1(1-3\alpha)s+1]^3} \tag{26}$$

and the control system transfer function for the disturbance has the form

$$G_{vy}(s) = \frac{Y(s)}{V(s)} = \frac{G_P(s)}{1+G_C(s)G_P(s)} = \frac{27k_1T_1^2(1-3\alpha)^3 s}{[3T_1(1-3\alpha)s+1]^3} \tag{27}$$

The both relations (26) and (27), likely as relations (21) – (25), for $\alpha = 0$ hold for PI controller ($r_1^* = 0,\ T_D^* = 0$)

From the relation (26) it follows that the nominator of the control system transfer function for $\alpha = 0.25$ (the value used by Ziegler and Nichols) has a stable double real zero and the corresponding PID controller adjustable parameters are given by the formulas

$$r_0^* = \frac{16}{3k_1T_1},\ T_I^* = \frac{9}{4}T_1,\ T_D^* = \frac{9}{16}T_1 \tag{28}$$

The servo and regulatory step responses for α = 0, 0.05, 0.1, 0.15, 0.2, 0.25 and k_1 = 1 are shown in Fig. 3.

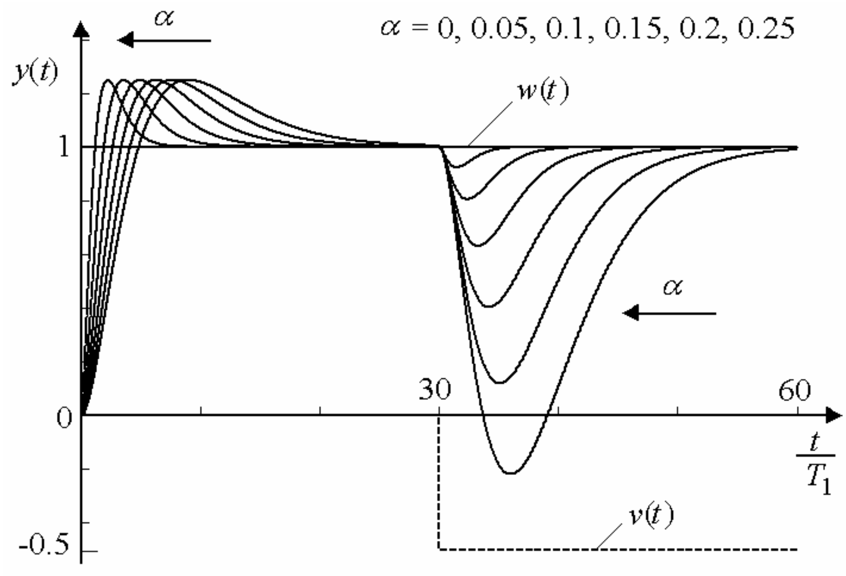

Fig. 3. Servo and regulatory step responses

From Fig. 3 it follows that the servo step responses for $0 \le \alpha \le 0.25$ show a practically constant overshoots about 25 %. For the value $\alpha = 0$ the PI controller is obtained and the tuning is rather conservative. For positive values $0 < \alpha \le 0.25$ PID controller is obtained. For the value $\alpha = 0.25$ the high-quality regulatory response is obtained. By the corresponding choice of value α, the controller tuning can conform to the limitation of the manipulated variable.

3 CONTROLLERS WITH TWO DEGREE OF FREEDOM

If the overshoot for the servo response is inadmissibly big, then it is possible to use the PID controller with two degree of freedom (2DOF), which is described by relation [Åström & Hägglund 2006; Vítečková, Víteček 2007, 2008]

$$U(s) = r_0 \left\{ bW(s) - Y(s) + \frac{1}{T_I s} E(s) + T_D s[cW(s) - Y(s)] \right\} \tag{29}$$

where b is the set-point weight of the proportional term, c – the set-point weight of the derivative term.

For $b = c = 1$ the standard PID controller (11) is obtained and for $b = 1$ a $T_D = 0$ the standard PI controller (13) is obtained.

The control system with the 2DOF controller (29) can be transformed in the system in Fig. 4 with the input filter with the transfer function

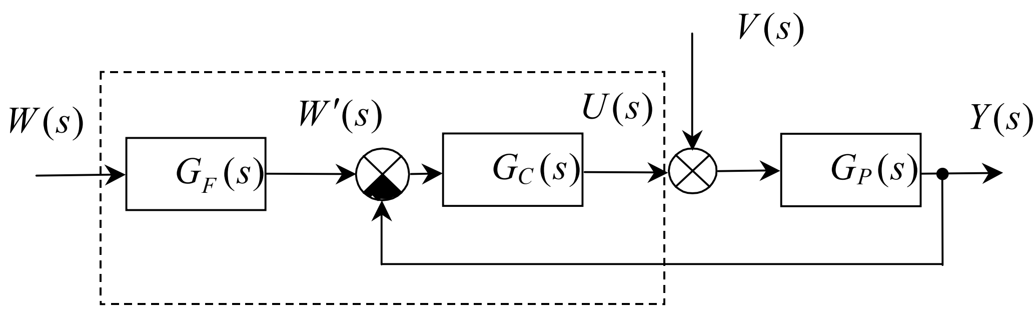

Fig. 4. Control system with 2DOF controller

$$G_F(s) = \frac{W'(s)}{W(s)} = \frac{cT_I T_D s^2 + bT_I s + 1}{T_I T_D s^2 + T_I s + 1} \tag{30}$$

and the PID standard controller with the transfer function (11).

For $T_D = 0$ from relation (29) the 2DOF PI controller can be obtained, which corresponds to the input filter in Fig. 4 with the transfer function

$$G_F(s) = \frac{W'(s)}{W(s)} = \frac{bT_I s + 1}{T_I s + 1} \tag{31}$$

and the standard PI controller with the transfer function (13).

For the 2DOF PID controller two poles of (26) can be compensated with two input filter zeros (30) by suitable choice of the set-point weights b and c. These weights can be obtained via comparison of the coefficients for the same power of the complex variable s, i.e.

$$\left(\frac{1}{|s_3^*|}s+1\right)^2 = [3T_1(1-3\alpha)s+1]^2 = cT_I^* T_D^* s^2 + bT_I^* s + 1 \tag{32}$$

After substitution (24) and (25) in the relation (32) the set-point weights

$$b = \frac{2}{3}, \qquad c = \frac{1}{9\alpha} \tag{33}$$

can be obtained.

Because $0 \le c \le 1$, the practical values of α for the 2DOF PID controller are

$$\frac{1}{9} \le \alpha \le \frac{1}{4} \tag{34}$$

Similarly for the 2DOF PI controller the set-point weight can be obtained from the relation

$$\frac{1}{|s_3^*|}s + 1 = 3T_1 s + 1 = bT_I^* s + 1 \tag{35}$$

It is obvious that is

$$b = \frac{1}{3} \tag{36}$$

The servo and regulatory step responses for α = 0.11, 0.15, 0.2, 0.25 and k_1 = 1 for 2DOF PID and 2DOF PI ($\alpha = 0$ and $b = 0.33$) controllers are shown in Fig. 5. The response for the 2DOF PI controller is very conservative.

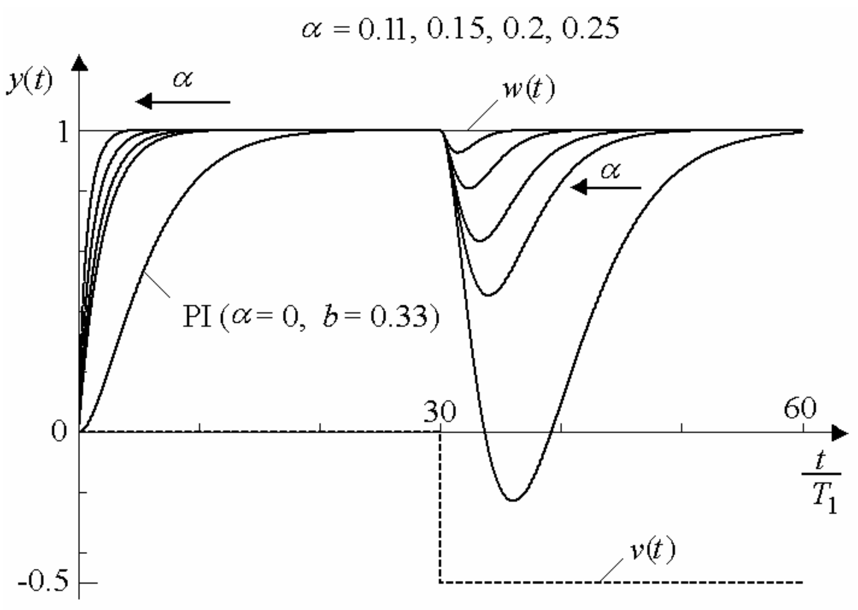

Fig. 5. Servo and regulatory responses for controllers with two degree of freedom

4 CONCLUSIONS

In this article the new controller tuning method for the first order plants with integration is derived, which comes from the multiple dominant pole method. The described method enables PI and PID controller tuning and by suitable choice of the derivative time to the integral time ratio to conform to disturbance rejection and to the limitation of the manipulated variables. For the 2DOF PI and PID controllers the set-point weights are derived.

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